Module 3

FIR filter design

FIR filter types

Types of FIR Filter

$h(n)$

- Symmetric
  - $h(n) = h(M-1-n)$
  - $\phi(\omega) = \text{phase} = -\left(\frac{M-1}{2}\right)\omega$
  - Even length (Type 2) (2, 1, 1, 2)
  - Odd (Type 1) (1, 2, 2, 2, 1)
- Antisymmetric
  - $h(n) = -h(-M-1-n)$
  - $\phi(\omega) = \frac{\pi}{2} - \left(\frac{M-1}{2}\right)\omega$
  - Even (Type 4)
  - Odd (Type 3)

Verify that type 1 FIR filter has linear phase response

Considering the sequence $h(n)$ of length 5

$\therefore M = 5$

$h(n) = \{h(0), h(1), h(2), h(3), h(4)\}$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=0}^{4} h(n) z^{-n}$$

$$= h(0) z^{0} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}$$

But from odd length symmetry ppt of Type 1 filter

~~$h(0) = h(4)$~~ $h(0)$ equivalent to $h(4)$ $h(1)$ to $h(3)$

$$= \frac{1}{z^2}\left[h(0)z^2 + h(4)z^{-2} + h(1)z + h(3)z^{-1} + h(2)\right]$$

replacing z by $e^{j\omega}$

$$= \frac{1}{(e^{j\omega})^2}\left[h(0)(e^{j\omega})^2 + h(4)e^{-2j\omega} + h(1)e^{j\omega} + h(3)e^{-j\omega} + h(2)\right]$$

$$= \frac{1}{e^{2j\omega}}\left[h(0)\left(e^{-j2\omega} + e^{j2\omega}\right) + h(1)\left[e^{j\omega} + e^{-j\omega}\right] + h(2)\right]$$

$$H(\omega) = e^{-2j\omega}\left[2h(0)\cos(2\omega) + 2h(1)\cos\omega + h(2)\right]$$

 AJN Notes

Show that FIR filter with Symmetric impulse response & even length will have compulsory zero at $z=-1$

consider $h(n)$ of length M

$$H(z) = \sum_{n=0}^{(M-1)} h(n) z^{-n} = \sum_{n=0}^{(M-1)} h(M-1-n) z^{-n}$$

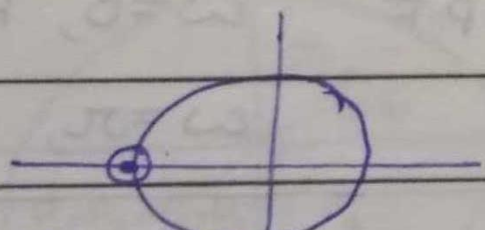
Replacing ~~h(n)~~ $M-1-n = m$

$$H(z) = \sum_{m=0}^{(M-1)} h(m) z^{-(M-1-m)}$$

 AJN Notes

$$H(z) = z^{-(m-1)} \sum_{m=0}^{(M-1)} h(m)(z^{-1})^{-m}$$

But $\boxed{H(z) = z^{\cancel{(m)(-1)}\,-(m-1)} H(z^{-1})}$



$z=-1$

$\omega = \pi$

$$z = e^{j\omega}$$

$$z = \cos(\omega) + j\sin(\omega)$$

$\omega = \pi$

$$z = e^{j\omega}$$

$$z = \cos(\omega) + j\sin(\omega)$$

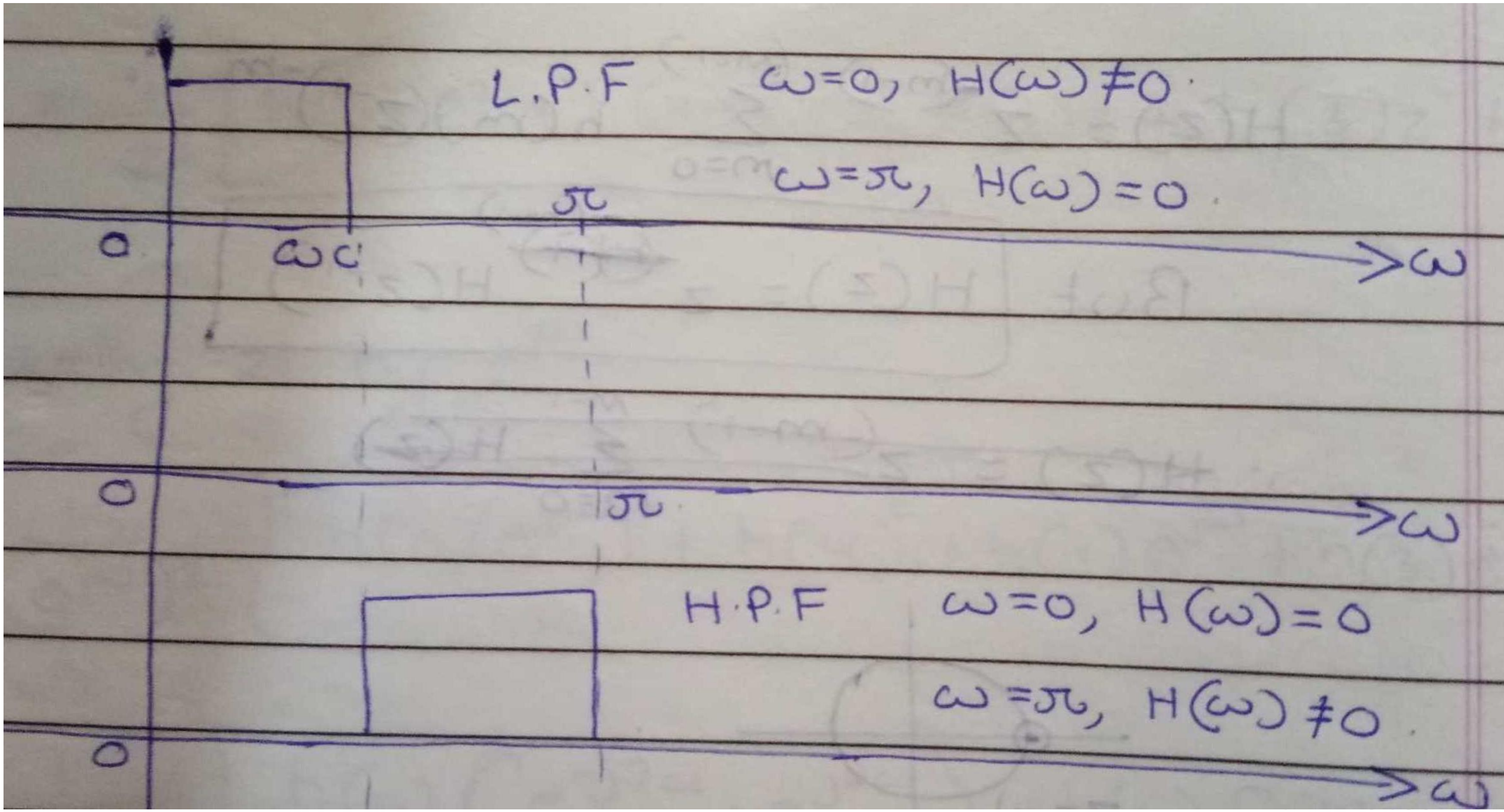
$$H(-1) = (-1)^{-(M-1)} h(-1)$$

$$H(-1) = (-1)\, H(-1)$$

$$2H(-1) = 0$$

$$H(-1) = 0$$

There is compulsory zero at $z = -1$



L.P.F $\omega = 0,\ H(\omega) \neq 0$.

$\omega = \pi,\ H(\omega) = 0$.

H.P.F $\omega = 0,\ H(\omega) = 0$

$\omega = \pi,\ H(\omega) \neq 0$.

118 Zero locations in linear phase FIR filters

Type 1	There is no compulsory zero	All types of filters can be designed
Type 2	compulsory zero at $z=-1$	L.P.F can be designed
Type 3	Compulsory zero at $z=-1$, $z=1$	B.P.F can be designed. But L.P & H.P and B.R cannot be designed
Type 4	Compulsory zero at $z=1$	H.P.F can be designed

AJN Notes

This image cannot currently be displayed.

This image cannot currently be displayed.

AJN Notes
AJN Notes
AJN Notes
AJN Notes
AJN Notes

Characteristics

If M is length of filter order is $N = M - 1$
zero's are always in reciprocal
If ~~these~~ zero's are complex & not on unit circle ~~they~~ then they will always occur in the pair of four
If the complex zero's are on unit circle then they will always occur in the pair of two

One of the zero of liner phase FIR filter is at $0.5\, e^{j\pi/3}$ show the location of other zeros and hence find the transfer function.

$$z_2 = \frac{1}{z_1}$$

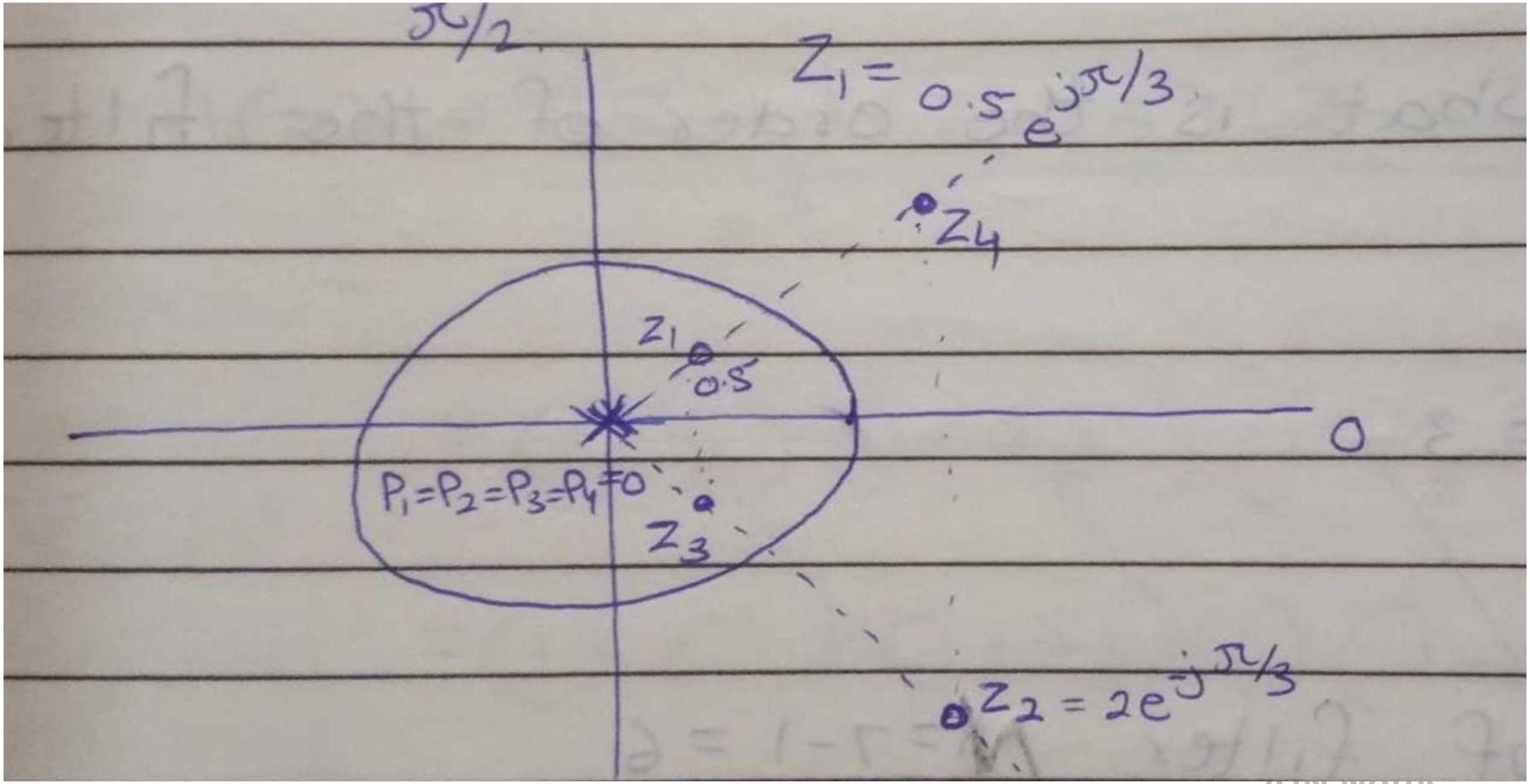
$$z_2 = 2e^{-j\pi/3}$$

$$z_3 = z_1^* = 0.5\,e^{-j\pi/3}.$$

$$z_4 = z_2^* = 2e^{j\pi/3}.$$

Number of zero's = Number of poles [For

= 4


π/2
Z1 = 0.5e^{jπ/3}
Z4
Z1
0.5
P1=P2=P3=P4=0
Z3
0
Z2 = 2e^{-jπ/3}

$$H(z) = \frac{(z-z_1)(z-z_2)(z-z_3)(z-z_4)}{z^4}$$

$$= \frac{(z-0.5e^{j\pi/3})(z-2e^{-j\pi/3})(z-0.5e^{-j\pi/3})(z-2e^{j\pi/3})}{z^4}$$

$$H(z) = (1-2.5z^{-1}) + \cancel{0.5z^{-2}}\, 5.25z^{-2} - 2.5z^{-3} + z^{-4}$$

→ No denominator

length of filter $M=5$ [odd length]

order of filter $N = 5-1 = 4$.

$$H(z) = \frac{(z-z_1)(z-z_2)(z-z_3)(z-z_4)}{z^4}$$

$$= \frac{(z-0.5e^{j\pi/3})(z-2e^{-j\pi/3})(z-0.5e^{-j\pi/3})(z-2e^{j\pi/3}}{z^4}$$

$$H(z) = (1-2.5z^{-1}) + \cancel{0.5z^{-2}}\ 5.25z^{-2} - 2.5z^{-3} + z^{-4}$$ → No denomi

length of filter $M=5$ [odd length]

order of filter $N = 5-1 = 4$.

$\{1, -2.5, 5.25, -2.5, 1\}$.

filter have odd length & even symmetry hence it is type 1 filter.

$H(\omega) = 2e^{-j3\omega}$, what is the order of the filter

Q) $H(\omega) = 2e^{-j3\omega}$ what is the order of the filter

$$\phi = -\left(\frac{M-1}{2}\right)\omega$$

ae^{jb}

$$\frac{M-1}{2} = 3$$

$$\therefore M = 7$$

Order of filter $N = 7 - 1 = 6$

$\therefore$ No of poles = No of zeros = 6

One of the zero of 3rd order FIR filter is at $z = 0.5$, the filter is symmetric. Find the T.F and the impulse response.

$N = 3 \longrightarrow$ Order

$\therefore M = 4 \rightarrow$ length [Even length]

Since it is even symmetric so type 2 filter

From the table the pole zero plot is,

AJN Notes

$Z_3 = \frac{1}{Z_2}$

$Z_3 = 2.$

Because order is 3 No of ze

poles = (

$\therefore H(z) = \frac{(z+1)(z-0.5)(z-2)}{z^3}$

AJN Notes

AJN Notes

AJN Notes

AJN Notes

$$= \frac{z^3 - z^2 - 2z - 0.5z^2 + 0.5z + 1}{z^3}$$

$$= \frac{z^3 - 1.5z^2 - 1.5z + 1}{z^3}$$

$$\boxed{H(z) = 1 - 1.5z^{-1} - 1.5z^{-2} + z^{-3}}$$

$h(n) = \{1, -1.5, -1.5, 1\}$ $\rightarrow$ $\sum_{-\infty}^{\infty} h(n)$

4 coefficients or samples

Design of linear phase FIR filters

Method 1 → Fourier Series

Method 2 → Windowing method

Method 3 → Frequency sampling

FIR filter design using fourier method

Assumptions

Given the desired freq response,

$H_d(\omega)$

(I) Calculate desired impulse ~~red~~ response $h_d(n)$ using inverse DTFT

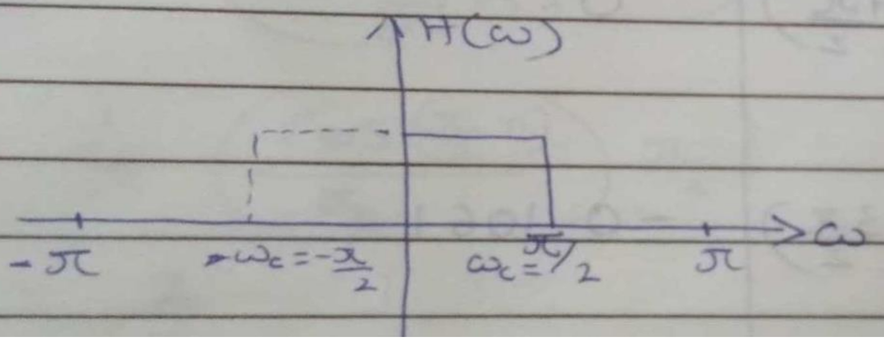
(II) Find ~~a~~ $h(n)$ By truncating $h_d(n)$ in the range $-\left(\frac{M-1}{2}\right) \leq n \leq \left(\frac{M-1}{2}\right)$ where M is number of tabs or coefficients of filter.

(III) Calculate non causal filter T.F $H_1(z)$ using z-transform of $h(n)$

(IV) Find the causal T.F $H(z)$ using,

$$H(z) = z^{-\left(\frac{M-1}{2}\right)} H_1(z)$$

Design FIR L.P.F with eleven tabs with cutoff frequency $\omega_c = \frac{\pi}{2}$



Desired impulse response

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)\, e^{+j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} (1)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{\pi n} \sin\left(\frac{\pi}{2} n\right)$$ → This in Valid for range $-\infty < n < \infty$

(II) $h(n) =$ truncation of $h_d(n)$

$M = 11$ i.e number of tabs

$$-\left(\frac{M-1}{2}\right) \le n \le \left(\frac{M-1}{2}\right)$$

$$-\left(\frac{11-1}{2}\right) \le n \le \left(\frac{11-1}{2}\right)$$

$$-5 \le n \le 5$$

n	h(n)
-5	$\frac{1}{-5\pi} \sin\left(-\frac{5\pi}{2}\right)$
-4	$\frac{1}{-4\pi} \sin\left(-\frac{4\pi}{2}\right)$
-3	$\frac{1}{-3\pi} \sin\left(-\frac{3\pi}{2}\right)$

N Notes

n	h(n)	o/p values
-5	$\frac{1}{-5\pi}\sin\left(\frac{-5\pi}{2}\right)$	$\cancel{8.699 \times 10^{-3}}$ 0.06366
-4	$\frac{1}{-4\pi}\sin\left(\frac{-4\pi}{2}\right)$	0
-3	$\frac{1}{-3\pi}\sin\left(\frac{-3\pi}{2}\right)$	-0.1061
-2	$\frac{1}{-2\pi}\sin\left(\frac{-2\pi}{2}\right)$	0

AJN Notes

Notes

AJN Notes

AJN Notes

-1	$\frac{1}{-\pi} \sin\left(-\frac{\pi}{2}\right)$	0.3183
0	$\cancel{\frac{-1}{0(\pi)} \sin\left(\frac{0\pi}{2}\right)}$	0.5 $\cancel{0}$ = $\cancel{\frac{1}{\pi n}}$ $\cancel{\frac{-1}{\pi n}}$
1	$\frac{1}{\pi} \sin\left(\frac{\pi}{2}\right)$	0.3183
2	$\frac{1}{2\pi} \sin\left(\frac{2\pi}{2}\right)$	0
3	$\frac{1}{3\pi} \sin\left(\frac{3\pi}{2}\right)$	-0.1061
4	$\frac{1}{4\pi} \sin\left(\frac{4\pi}{2}\right)$	0
5	$\frac{1}{5\pi} \sin\left(\frac{5\pi}{2}\right)$	0.06366

-1	$\frac{1}{-\pi} \sin\left(-\frac{\pi}{2}\right)$
0	$\cancel{\frac{-1}{0(\pi)} \sin\left(\frac{0\pi}{2}\right)}$
1	$\frac{1}{\pi} \sin\left(\frac{\pi}{2}\right)$
2	$\frac{1}{2\pi} \sin\left(\frac{2\pi}{2}\right)$
3	$\frac{1}{3\pi} \sin\left(\frac{3\pi}{2}\right)$
4	$\frac{1}{4\pi} \sin\left(\frac{4\pi}{2}\right)$
5	$\frac{1}{5\pi} \sin\left(\frac{5\pi}{2}\right)$

IV) Find the causal T.F $H(z)$ using,

$$H(z) = z^{-\left(\frac{M-1}{2}\right)} H_1(z)$$

$$H(z) = z^{-5} \sum_{n=-5}^{5} h(n) z^{-n}$$

$$= 0.0636 - 0.1061 z^{-2} + 0.3183 z^{-4} + 0.5 z^{-5} + 0.3183 z^{-6} - 0.1061 z^{-8} + 0.0636 z^{-10}$$

It is causal filter since power of z is starting at zero & is negative so on.

$h(n) = \{0.0636, 0, -0.1061, 0, 0.3183, 0.5, 0.3183, 0, -0.1061, 0, 0.0636\}$

odd length, even symmetry ∴ Type 1 filter.

 AJN Notes